



# Applications of Quantum Annealing in Computational Finance

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D-Wave User Conference, Santa Fe, Sept. 2016

# Outline

- Where's my Babel Fish?
- Quantum-Ready Applications for Computational Finance
- Tools and Resources

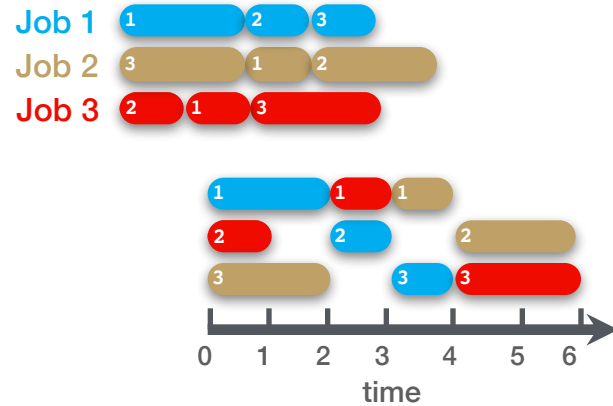


**Where's my Babel Fish?**

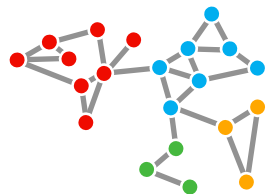


# Where's my Babel Fish?

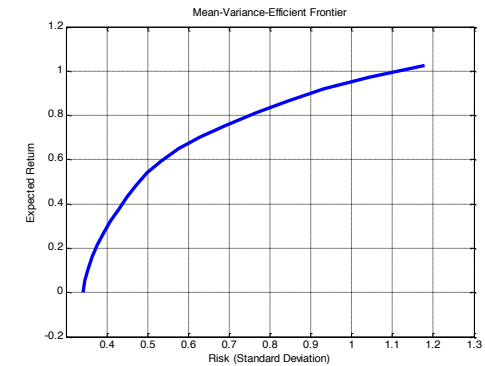
- k-Coloring
- Connected Dominating Set
- Job Shop Scheduling



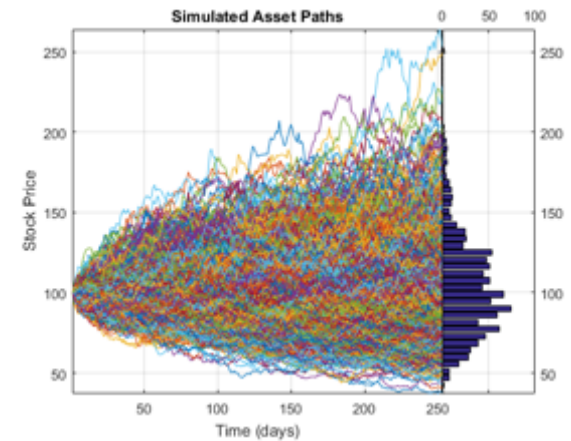
- Graph Similarity
- Graph Partitioning



- Portfolio Optimization



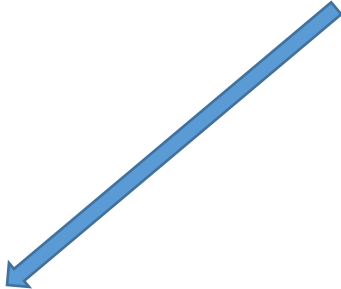
- Asset Allocation
- Risk Management

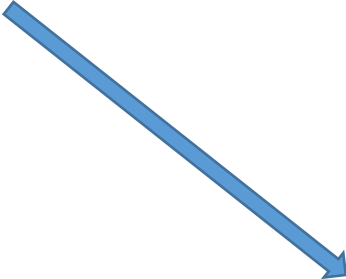


- Option Pricing

# Is that a QUBO?

$$\min_x (x^T Q x)$$


$$x \in \{0,1\}^N$$
$$Q \in \mathbb{R}^{N \times N}$$


$$x \in \mathbb{R}^N$$
$$Q \in \mathbb{R}^{N \times N}$$

Weight of assets  
in portfolio



Covariance  
matrix





# Quantum-Ready Applications for Computational Finance

# Applications for Computational Finance

- A Multi-Period Optimal Trading Strategy
- Quantum-Ready Hierarchical Risk Parity (QHRP)
- Real-Time Optimization Framework
- Tax Loss Harvesting



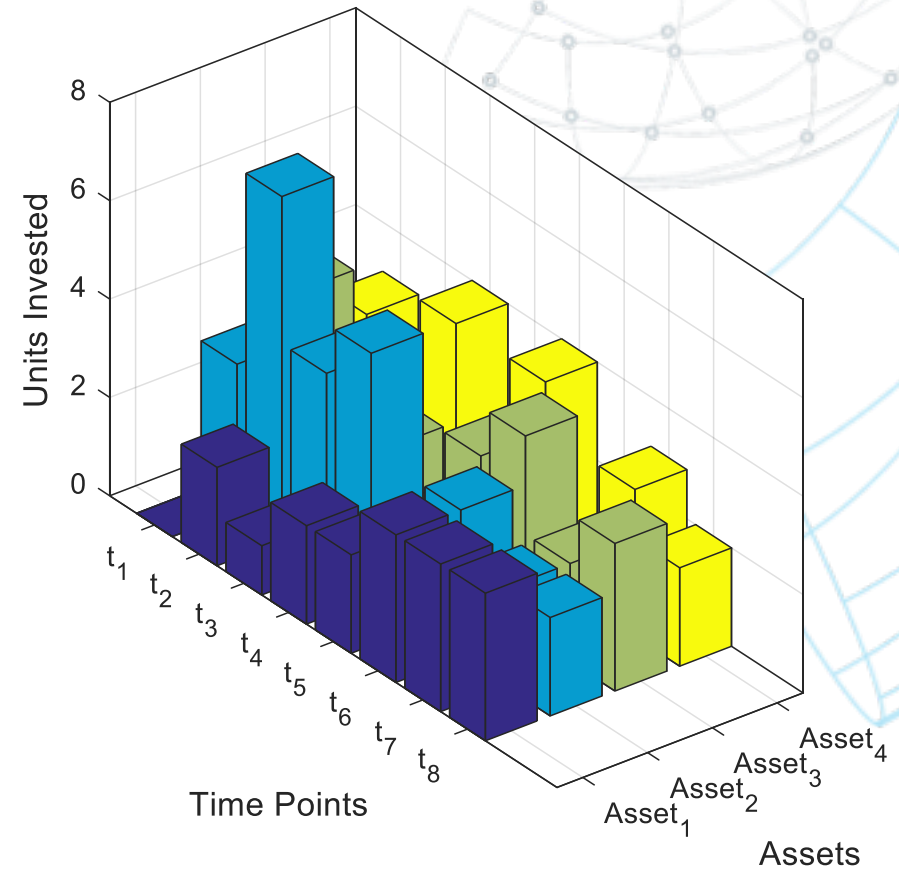


# Optimal Trading Trajectories



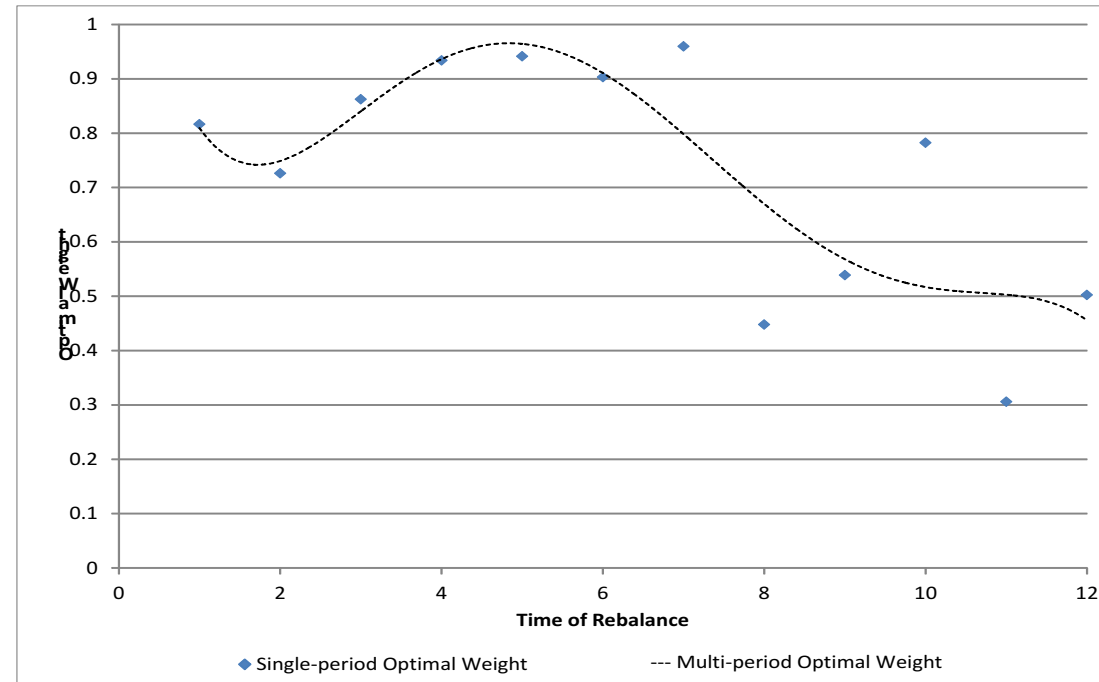
# A Multi-Period Optimal Trading Strategy

- The optimal trading trajectory problem
- The mathematical formulation
- Considerations in using the quantum annealer
- Experimental Results



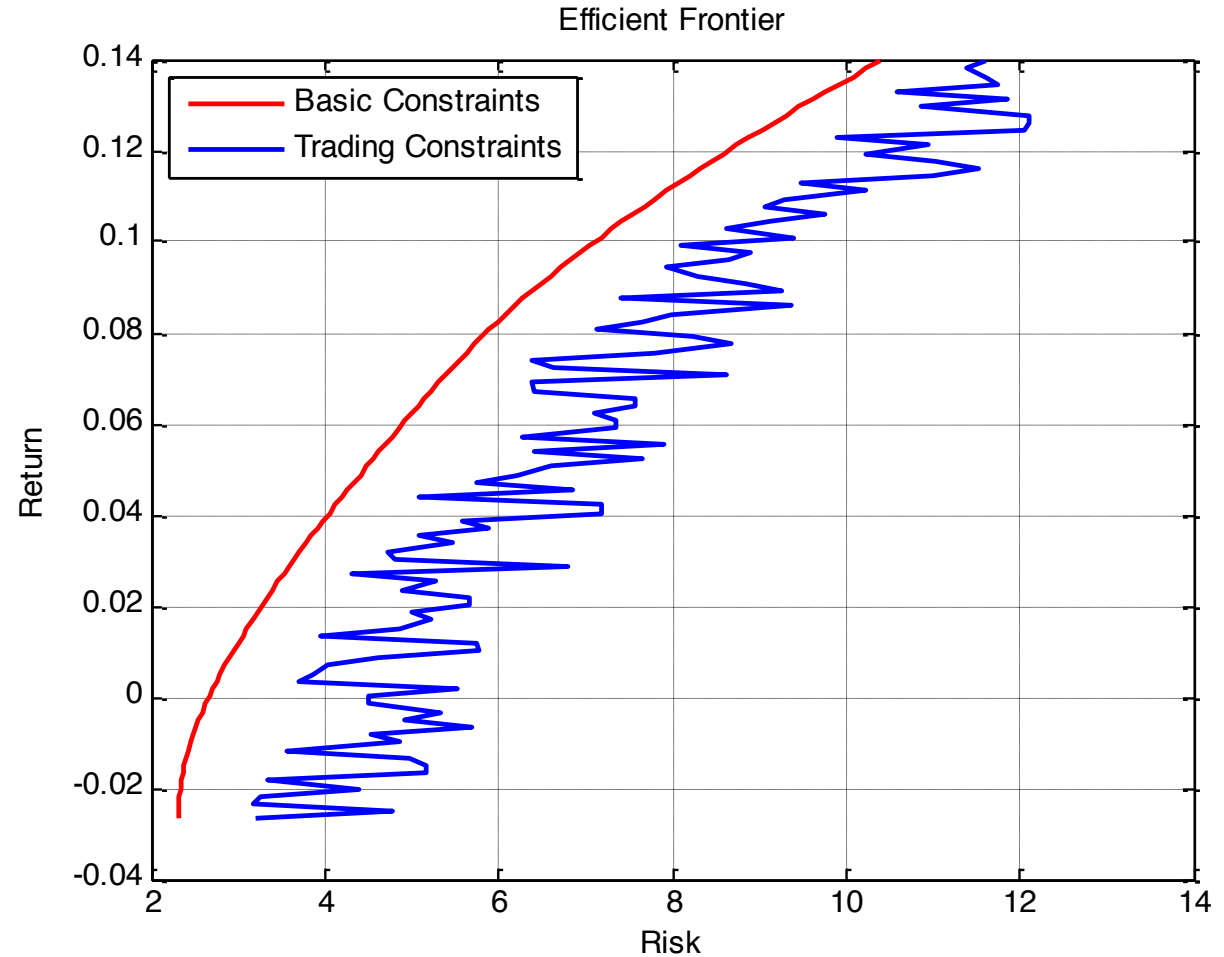
# The Optimal Trading Trajectory Problem

- Managers of large portfolios typically need to optimize their portfolios over a multiple period horizon.
- A sequence of single-period optimal positions is rarely multi-period optimal.
- Rebalancing the portfolio to align with each single period optimal weight is typically prohibitively expensive.



# Practical Considerations

- **Friction:** Transaction costs prevent portfolio managers from monetizing much of their forecasting power
- **Market Impact:** The sale or purchase of large blocks of a given asset may result in temporary and/or permanent price movements
- **Constraints:** Some positions can only be traded in blocks (e.g. real-estate, private offerings, fixed lot sizes,...), thus requiring integer solutions



# Mathematical Formulation

- The multi-period integer optimization problem may be written as

$$w = \operatorname{argmax}_w \left\{ \sum_{t=1}^T \left( \underbrace{\mu_t^T w_t}_{\text{returns}} - \frac{\gamma}{2} \underbrace{w_t^T \Sigma_t w_t}_{\text{risk}} - \underbrace{\Delta w_t^T \Lambda_t \Delta w_t}_{\substack{\text{direct costs,} \\ \text{temp. impact}}} - \underbrace{\Delta w_t^T \Lambda'_t \Delta w_t}_{\text{perm. impact}} \right) \right\}$$

s.t.:

mean-variance portfolio optimization

$$\forall t: \sum_{n=1}^N w_{n,t} \leq K ; \forall t, \forall n: w_{n,t} \leq K'$$

transaction cost and market impacts

- To solve, the problem must be converted to standard QUBO form:

$$\min_x x^T Q x$$

$$x \in \{0,1\}^N, Q \in \mathbb{R}^{N \times N}$$

# Bit Encoding

- The integer variables of the optimization problem ( $w_i$ ) must be recast to the binary variables used by the annealer ( $x_i$ ).

TABLE II: Comparison of the four encodings described in Section II-B. The column “Variables” refers to the number of binary variables required to represent a particular problem. The column “Largest integer” refers to a worst-case estimate of the largest integer that could be represented based on the limitation imposed by the noise level  $\epsilon$  and the ratio between the largest and smallest problem coefficients  $\delta$  and  $n \equiv 1/\sqrt{\epsilon\delta}$ .

Encoding	Variables	Largest integer	Notes
Binary	$TN \log_2(K' + 1)$	$\lfloor 2n \rfloor - 1$	Most efficient in number of variables; allows representing of the second-lowest integer.
Unary	$TNK'$	No limit	Biases the quantum annealer due to differing redundancy of code words for each value; encoding coefficients are even, giving no dependence on noise, so it allows representing of the largest integer.
Sequential	$\frac{1}{2}TN (\sqrt{1 + 8K'} - 1)$	$\frac{1}{2} \lfloor n \rfloor (\lfloor n \rfloor + 1)$	Biases the quantum annealer (but less than unary encoding); second-most-efficient in number of variables; allows representing of the second-largest integer.
Partition	$\leq T \binom{K+N-1}{N-1}$	$\lfloor n \rfloor$	Can incorporate complicated constraints easily; least efficient in number of variables; only applicable for problems in which groups of variables are required to sum to a constant; allows representing the lowest integer.

# Experimental Procedure and Results

- Generate a random problem: number of assets; time horizon; and total investable assets
- Solve using quantum annealer
- Find exact minimum solution using an exhaustive search
- Evaluate performance by how far the quantum solution is from the exact solution

TABLE IV: Results using D-Wave’s quantum annealer (with 200 instances per problem):  $N$  is the number of assets,  $T$  is the number of time steps,  $K$  is the number of units to be allocated at each time step and the maximum allowed holding (with  $K' = K$ ), “encoding” refers to the method of encoding the integer problem into binary variables, “vars” is the number of binary variables required to encode the given problem, “density” is the density of the quadratic couplers, “qubits” is the number of physical qubits that were used, “chain” is the maximum number of physical qubits identified with a single binary variable, and  $S(\alpha)$  refers to the success rate given a perturbation magnitude  $\alpha\%$  (explained in the text).

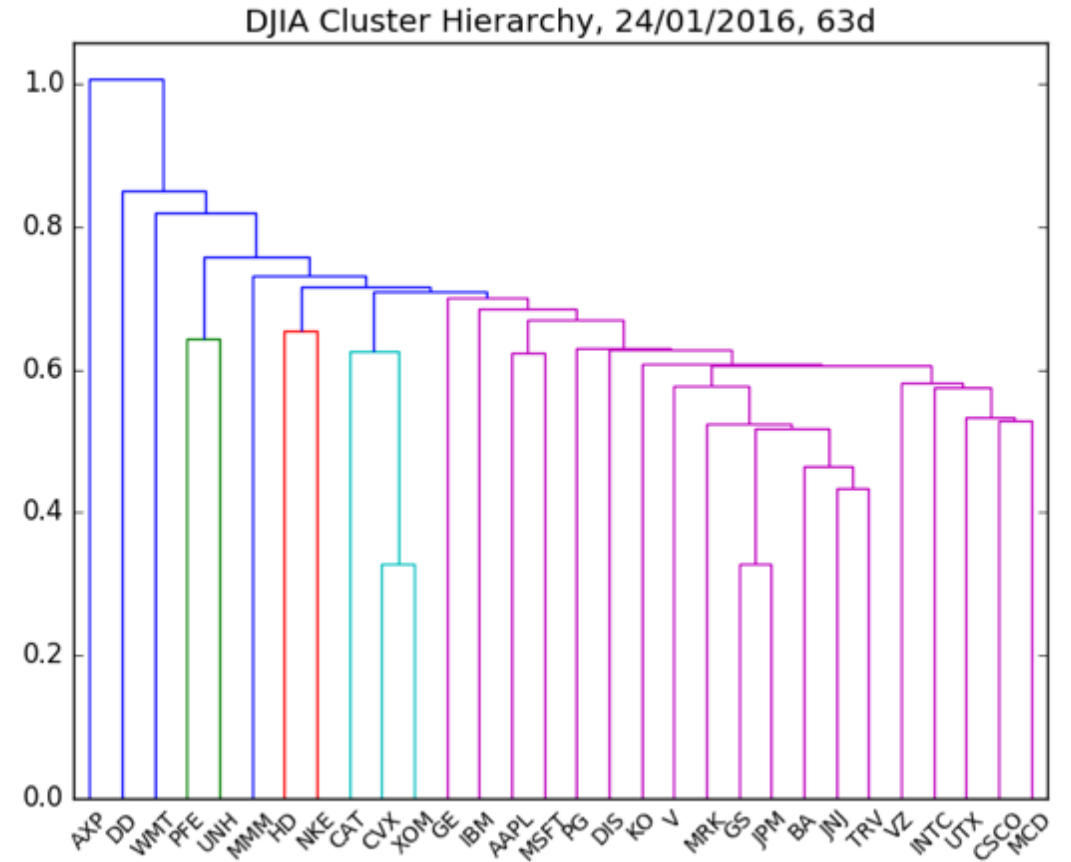
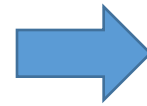
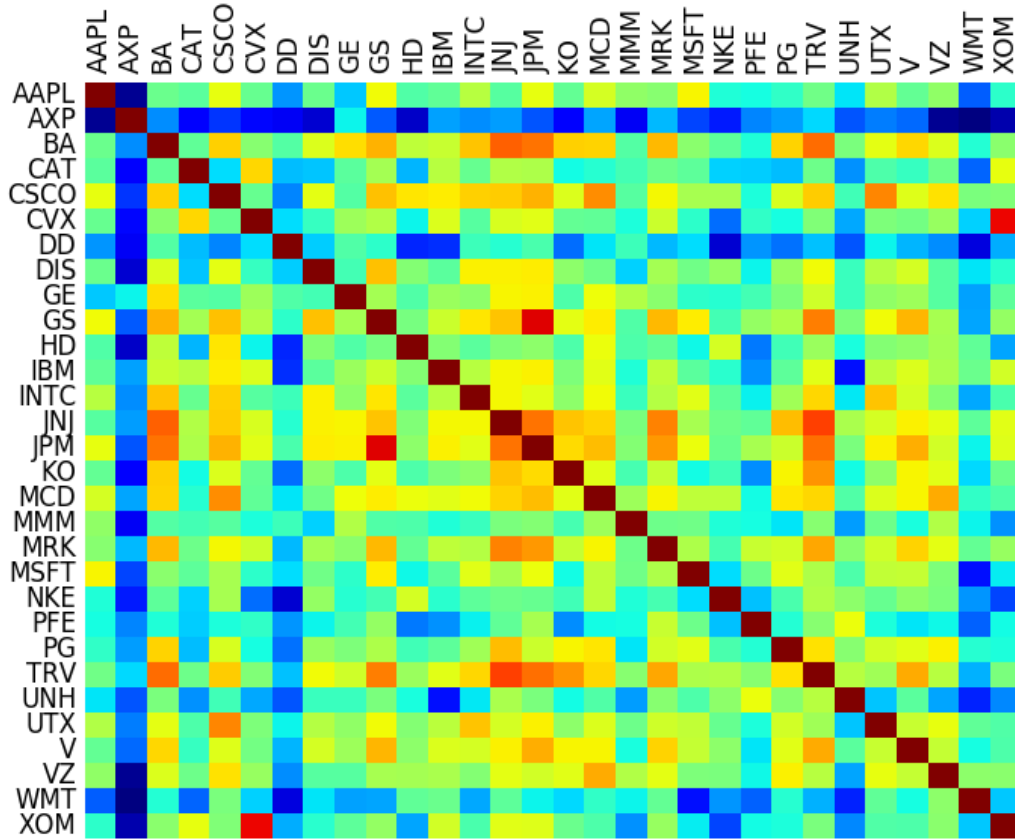
$N$	$T$	$K$	encoding	vars	density	qubits	chain	$S(0)$	$S(1)$	$S(2)$
2	3	3	binary	12	0.52	31	3	100.00	100.00	100.00
2	2	3	unary	12	0.73	45	4	97.00	100.00	100.00
2	4	3	binary	16	0.40	52	4	96.00	100.00	100.00
2	3	3	unary	18	0.53	76	5	94.50	100.00	100.00
2	2	7	binary	12	0.73	38	4	90.50	100.00	100.00
2	5	3	binary	20	0.33	63	4	89.00	100.00	100.00
2	6	3	binary	24	0.28	74	4	50.00	100.00	100.00
3	2	3	unary	18	0.65	91	6	38.50	80.50	95.50
3	3	3	binary	18	0.45	84	5	35.50	80.50	96.50
3	4	3	binary	24	0.35	106	6	9.50	89.50	100.00

- The annealer solution is typically within a small and acceptable margin of error of the exact globally minimal solution.



# Quantum-Ready Hierarchical Risk Parity

# Quantum-Ready Hierarchical Risk Parity



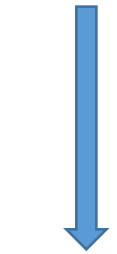


# The Approach

- Use tree structure to reduce connections between assets as well as estimation error
- Instead of minimizing variance via all correlations using all assets at once, solve the minimum variance problem at each node of the tree
- Build the final weight vector by recursively minimizing variance from the bottom of the tree to the top
- Ignore correlations in determining weights, use only in calculating variance of sub-trees
- Effect: Improve out-of-sample realized volatility
- Can also be applied to linear regression

$$\mathbf{w}^* = \arg \min_{\mathbf{w}} \mathbf{w}^T \hat{\Sigma} \mathbf{w} \quad s. t. \sum_i w_i = 1$$

Classical

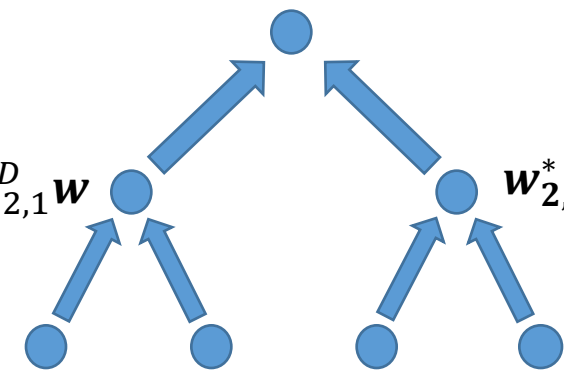


QHRP

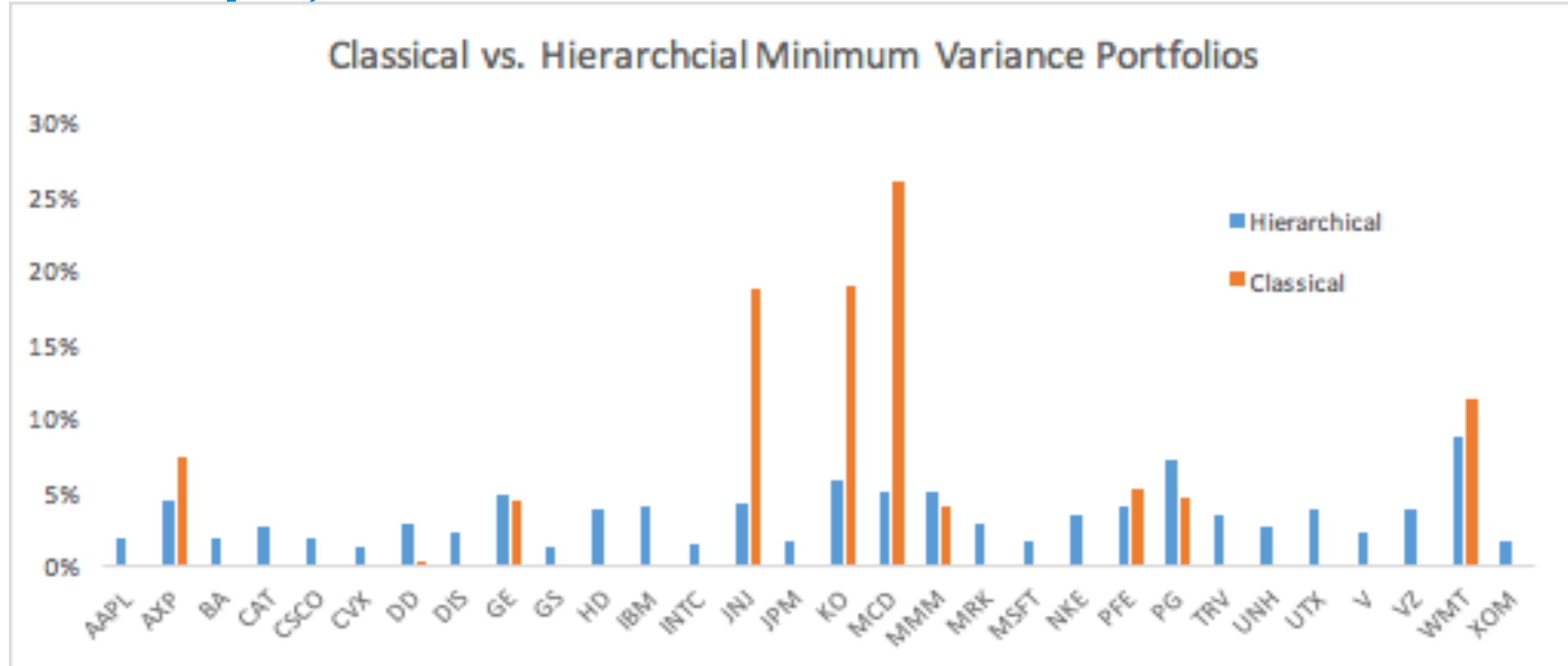
$$\mathbf{w}_{1,1}^* = \arg \min_{\mathbf{w}} \mathbf{w}^T \Sigma_{1,1}^D \mathbf{w}$$

$$\mathbf{w}_{2,1}^* = \arg \min_{\mathbf{w}} \mathbf{w}^T \Sigma_{2,1}^D \mathbf{w}$$

$$\mathbf{w}_{2,2}^* = \arg \min_{\mathbf{w}} \mathbf{w}^T \Sigma_{2,2}^D \mathbf{w}$$



# (Out-of-Sample) Minimum Risk Portfolios



- Classical algorithm minimizes **in-sample** risk, but sometimes missed the point
- In this example, it invests over 50% of the portfolio in just McDonalds, Coca-Cola and Johnson & Johnson
- QHRP provides more diversification



20%



## Risk Improvement in Simulations

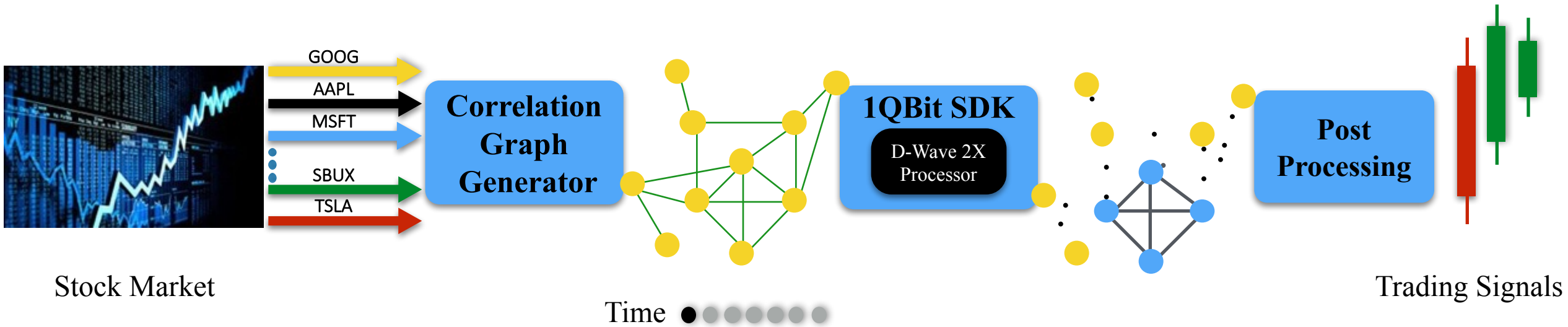
Out-of-sample volatility is reduced by 20% in simulated examples using 10 assets with 10% volatility each, random shocks, random correlations.



# Real-Time Optimization Framework

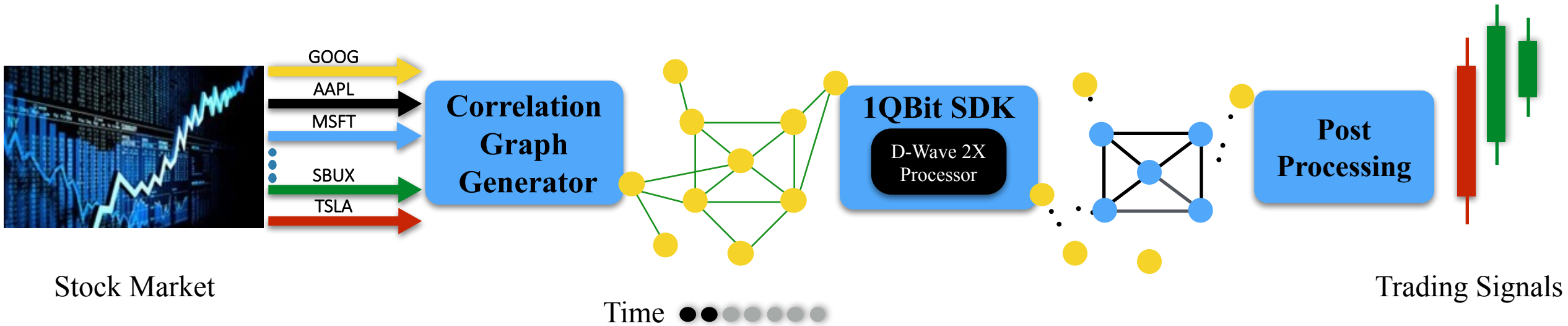
# Real-Time Optimization Framework

- Stream Analytics
- Real-Time data signals
- Quantum powered analytics



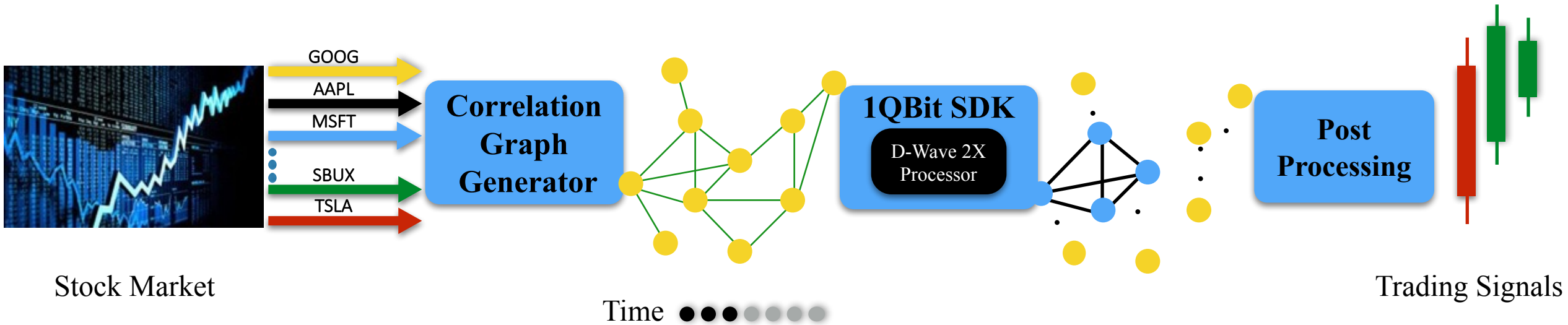
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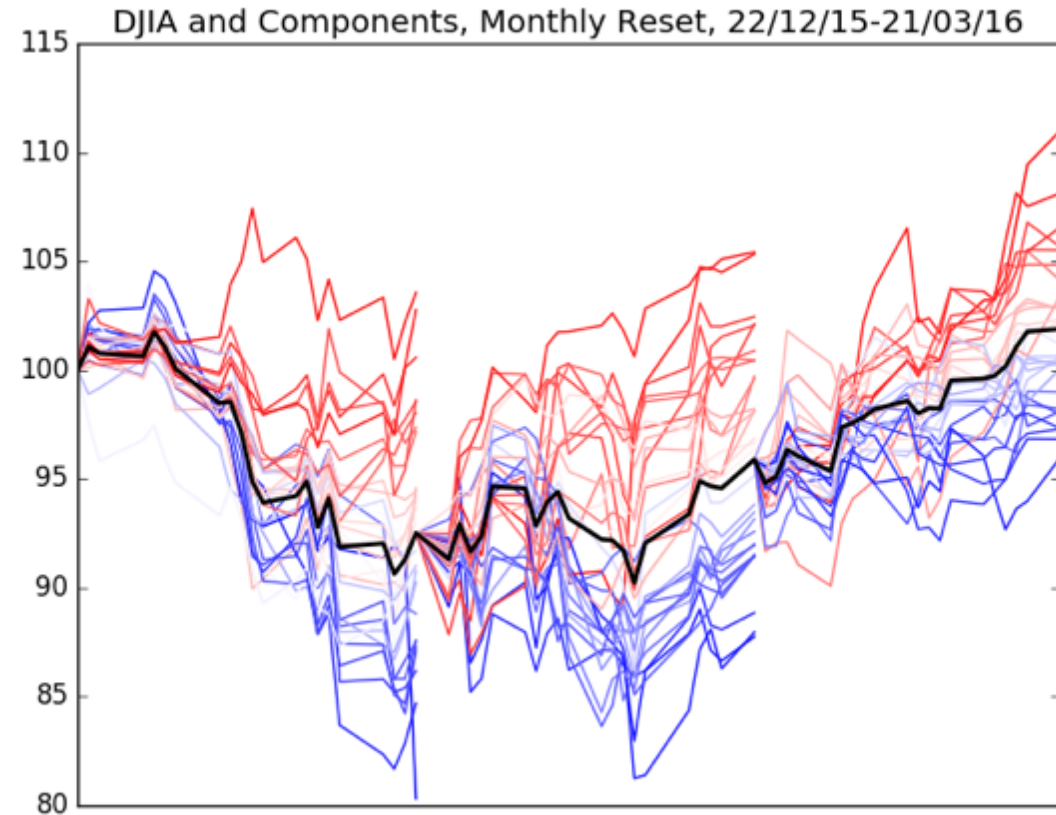


# Tax Loss Harvesting



# Tax Loss Harvesting

- Track an index using a subset of the components of the index while selling losses and moving into unowned components
- Integer Quadratic Programming: sell quantities from owned lots; buy new lots (avoiding wash sales)
- Offset gains and income; carry losses forward; create a tax buffer
- Quantum annealer used to find the optimal share quantities to buy and sell to generate the greatest tax benefit while staying within a specified tracking error.



# Tools and Resources

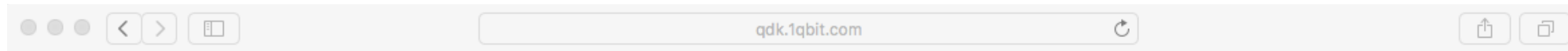


# Tools

## 1QBit's quantum-ready Software Development Kit:

Explore: [1qbit.com](https://1qbit.com)

Register for Beta Release: [qdk.1qbit.com](https://qdk.1qbit.com)



# 1QBit

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### NOTEBOOK

Run the tutorials and explore the functionalities provided by 1QBit's SDK in an interactive Jupyter Notebook.

[LOAD](#)



### TUTORIALS

Multiple tutorials with code samples are available that explain various features, including application development using the SDK's Algorithms layer.

[REVIEW](#)



### DOCUMENTATION

Extensive documentation can be accessed including chapters, code samples, and a reference manual of sample functions and classes in 1QBit's SDK.

[READ MORE](#)

The 1QBit logo, featuring the text "1QBit" in a bold, blue, sans-serif font.

# Resources: [www.quantumforquants.org](http://www.quantumforquants.org)

The screenshot shows the homepage of the Quantum for Quants website. The browser address bar displays [www.quantumforquants.org](http://www.quantumforquants.org). The website header includes the logo, a search bar, and navigation links for Home, Forum, Mission, Quantum Computing, Resources, Open Challenges, and Blog. A 'Log in or Register' link is also present. The main content area features a large banner with the text 'Welcome to Quantum for Quants' and a paragraph explaining the site's purpose: 'Quantum computers have the potential to provide an entirely different approach to solving very hard finance problems. Quantumforquants.org has been created to foster education, discussion, and collaboration to advance our identification and understanding of solutions to open industry problems.' Below the banner are three columns: 'Mission' (Foster education, discussion, and industry collaboration on open industry problems. >Learn more.), 'Quantum Computing' (Quantum computers have the potential to address very hard problems in an entirely different way - letting "physics solve math problems". >Learn more.), and 'New Challenges' (Watch this space for new challenges. >Learn more.). The left sidebar contains a '+ ASK A QUESTION' button, 'Recent Posts' (A Finance at the service of society, Q&A on Financial Quantum Computing with Marcos Lopez de Prado, Redefining problems with quantum computers, Why Quantum Finance?), 'Archives' (May 2016, April 2016), and 'Categories' (Quantum Computing, Tools, Uncategorized). The right sidebar is titled 'Latest Questions' and lists several questions, including 'Presentation of myself + Question about future studies', 'Complexity Classes: Are we on the cusp of change?', 'Extracting binary variables from economic data: Is there a quantum view?', 'Can a binary graph model be used to model information flow in CDS trading?', 'Can clique analysis tell us something new about time series?', 'How does QC help apply machine learning techniques?', 'What is the best academic background for learning about QC?', and 'Is bank account...'. The bottom section is titled 'From the Blog' and shows a post titled 'A Finance at the service of society' with a thumbnail image of a dome structure.

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+ ASK A QUESTION

## Welcome to Quantum for Quants

Quantum computers have the potential to provide an entirely different approach to solving very hard finance problems. Quantumforquants.org has been created to foster education, discussion, and collaboration to advance our identification and understanding of solutions to open industry problems.

### Mission

Foster education, discussion, and industry collaboration on open industry problems. >Learn more.

### Quantum Computing

Quantum computers have the potential to address very hard problems in an entirely different way - letting "physics solve math problems". >Learn more.

### New Challenges

Watch this space for new challenges. >Learn more.

## Recent Posts

- A Finance at the service of society
- Q&A on Financial Quantum Computing with Marcos Lopez de Prado
- Redefining problems with quantum computers
- Why Quantum Finance?

## Archives

- May 2016
- April 2016

## Categories

- Quantum Computing
- Tools
- Uncategorized

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## From the Blog

### A Finance at the service of society

For Finance to serve society, it

### Redefining problems with quantum computers



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