Applications of Quantum Annealing in Computational Finance

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D-Wave User Conference, Santa Fe, Sept. 2016

Outline

- Where’s my Babel Fish?
- Quantum-Ready Applications for Computational Finance
- Tools and Resources

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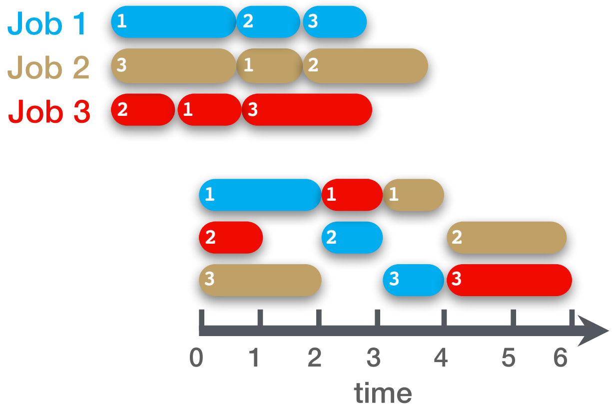
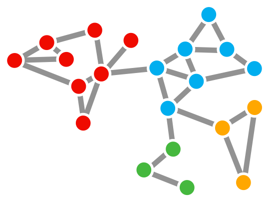
Where’s my Babel Fish?

Where’s my Babel Fish?

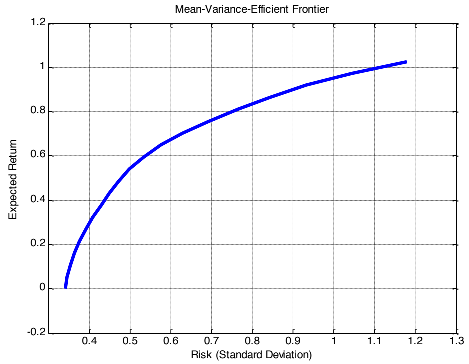
- k-Coloring
- Connected Dominating Set
- Job Shop Scheduling



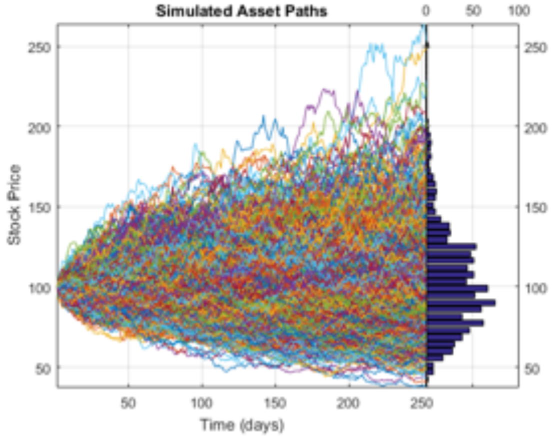
- Graph Similarity
- Graph Partitioning



- Portfolio Optimization



- Asset Allocation
- Risk Management



- Option Pricing

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Is that a QUBO?

$$\min_x (x^T Q x)$$

$$x \in \{0,1\}^N$$
$$Q \in \mathbb{R}^{N \times N}$$

$$x \in \mathbb{R}^N$$
$$Q \in \mathbb{R}^{N \times N}$$

Weight of assets in portfolio

Covariance matrix

Quantum-Ready Applications for Computational Finance

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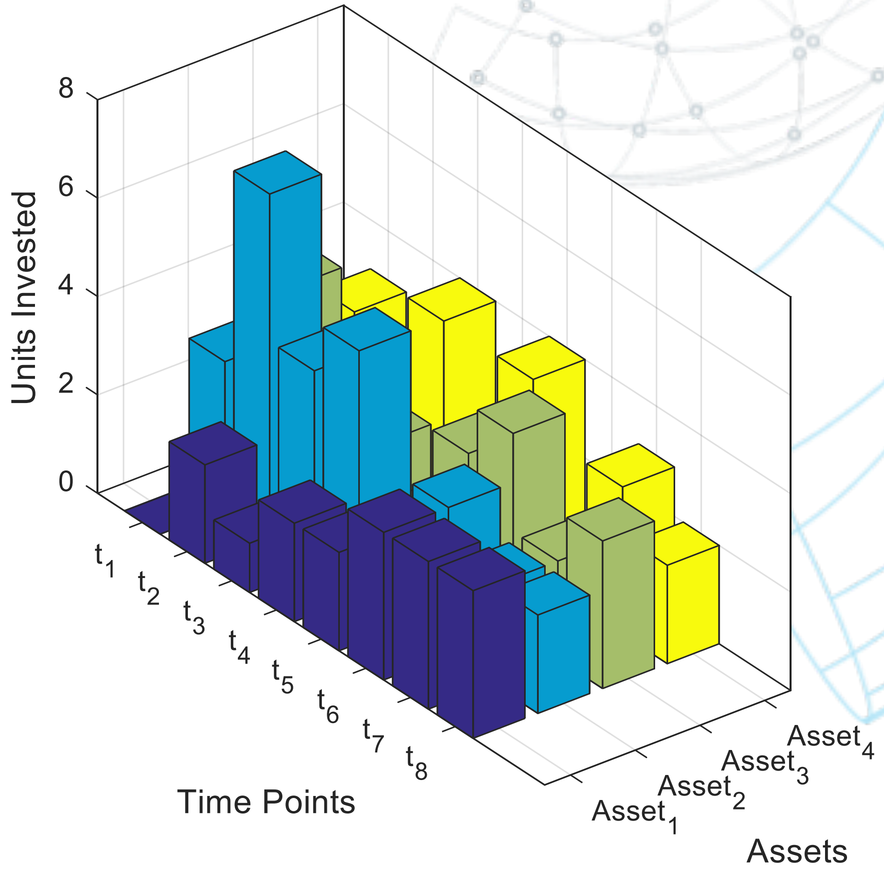
Applications for Computational Finance

- A Multi-Period Optimal Trading Strategy
- Quantum-Ready Hierarchical Risk Parity (QHRP)
- Real-Time Optimization Framework
- Tax Loss Harvesting

Optimal Trading Trajectories

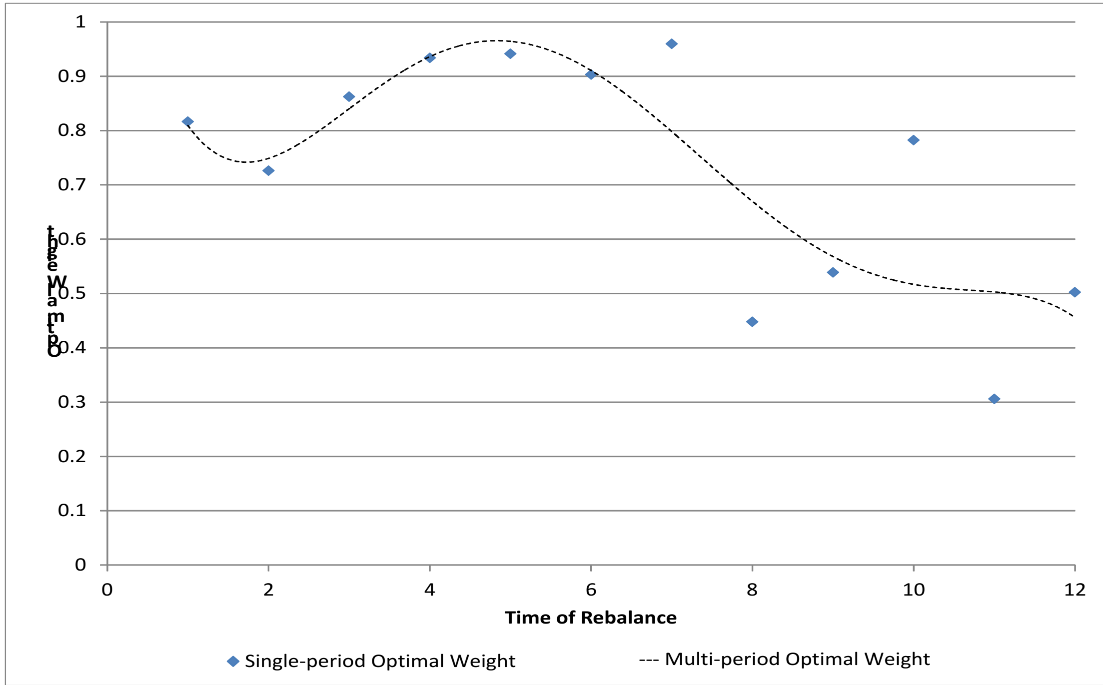
A Multi-Period Optimal Trading Strategy

- The optimal trading trajectory problem
- The mathematical formulation
- Considerations in using the quantum annealer
- Experimental Results



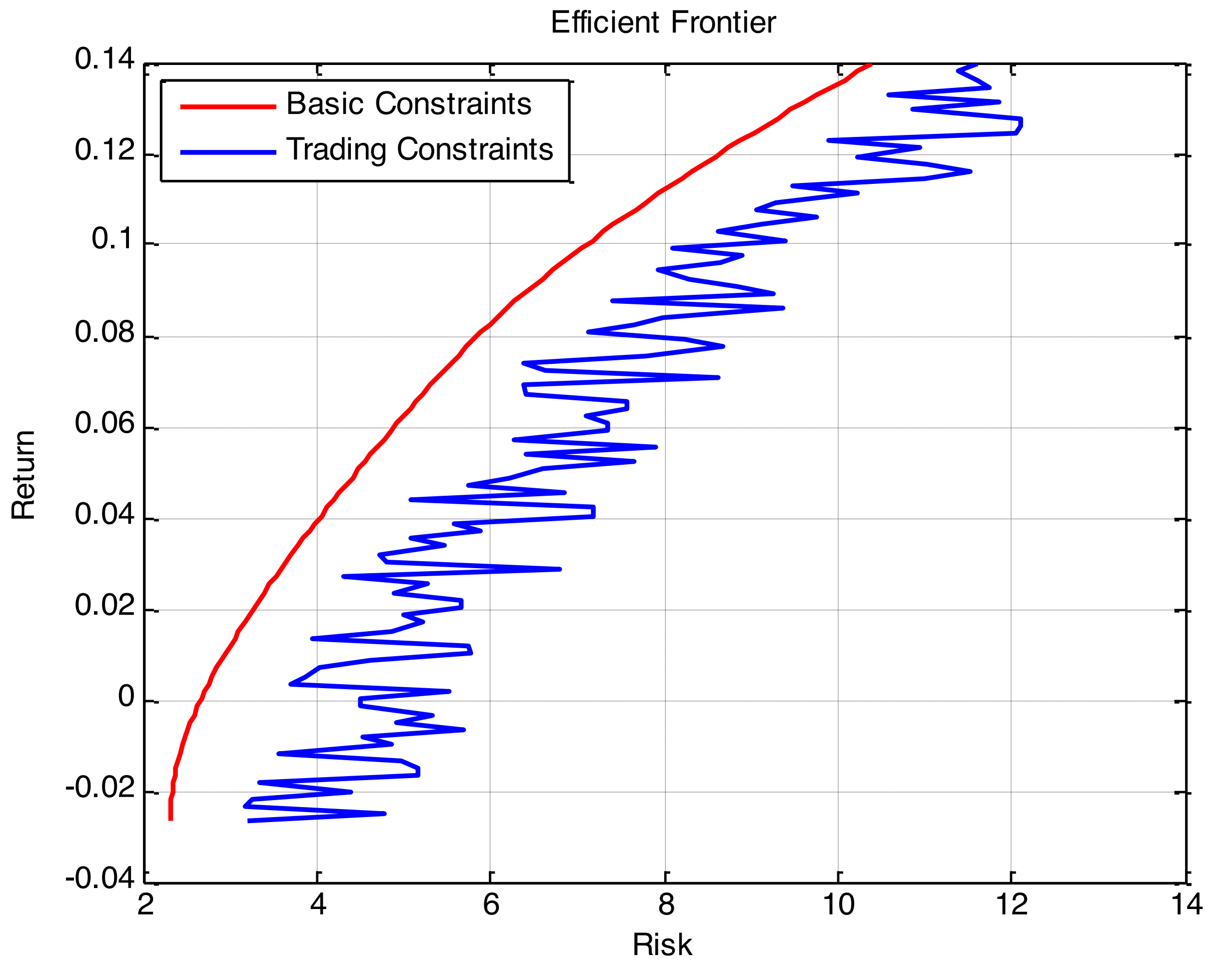
The Optimal Trading Trajectory Problem

- Managers of large portfolios typically need to optimize their portfolios over a multiple period horizon.
- A sequence of single-period optimal positons is rarely multi-period optimal.
- Rebalancing the portfolio to align with each single period optimal weight is typically prohibitively expensive.



Practical Considerations

- **Friction**: Transaction costs prevent portfolio managers from monetizing much of their forecasting power
- **Market Impact**: The sale or purchase of large blocks of a given asset may result in temporary and/or permanent price movements
- **Constraints**: Some positions can only be traded in blocks (e.g. real-estate, private offerings, fixed lot sizes,…), thus requiring integer solutions



Mathematical Formulation

- The multi-period integer optimization problem may be written as

$$w = \operatorname{argmax}_w \left\{ \sum_{t=1}^{T} \left(\underbrace{\mu_t^T w_t}_{\text{returns}} - \frac{\gamma}{2} \underbrace{w_t^T \Sigma_t w_t}_{\text{risk}} - \underbrace{\Delta w_t^T \Lambda_t \Delta w_t}_{\substack{\text{direct costs,}\\ \text{temp.impact}}} - \underbrace{\Delta w_t^T \Lambda_t' \Delta w_t}_{\text{perm.impact}} \right) \right\}$$

s.t.:

mean-variance
portfolio optimization

$$\forall t\colon \sum_{n=1}^{N} w_{n,t} \leq K \,; \forall t, \forall n\colon w_{n,t} \leq K'$$

transaction cost
and market impacts

- To solve, the problem must be converted to standard QUBO form:

$$\min_x x^T Q x$$
$$x \in \{0,1\}^N, Q \in \mathbb{R}^{N \times N}$$

Bit Encoding

- The integer variables of the optimization problem (w_i) must be recast to the binary variables used by the annealer (x_i).

TABLE II: Comparison of the four encodings described in Section II-B. The column "Variables" refers to the number of binary variables required to represent a particular problem. The column "Largest integer" refers to a worst-case estimate of the largest integer that could be represented based on the limitation imposed by the noise level ϵ and the ratio between the largest and smallest problem coefficients δ and $n \equiv 1/\sqrt{\epsilon\delta}$.

Encoding	Variables	Largest integer	Notes
Binary	$TN\log_2(K'+1)$	$\lfloor 2n \rfloor - 1$	Most efficient in number of variables; allows representing of the second-lowest integer.
Unary	TNK'	No limit	Biases the quantum annealer due to differing redundancy of code words for each value; encoding coefficients are even, giving no dependence on noise, so it allows representing of the largest integer.
Sequential	$\frac{1}{2}TN\left(\sqrt{1+8K'}-1\right)$	$\frac{1}{2}\lfloor n \rfloor \left(\lfloor n \rfloor + 1\right)$	Biases the quantum annealer (but less than unary encoding); second-most-efficient in number of variables; allows representing of the second-largest integer.
Partition	$\leq T\binom{K+N-1}{N-1}$	$\lfloor n \rfloor$	Can incorporate complicated constraints easily; least efficient in number of variables; only applicable for problems in which groups of variables are required to sum to a constant; allows representing the lowest integer.

Experimental Procedure and Results

- Generate a random problem: number of assets; time horizon; and total investable assets
- Solve using quantum annealer
- Find exact minimum solution using an exhaustive search
- Evaluate performance by how far the quantum solution is from the exact solution

TABLE IV: Results using D-Wave's quantum annealer (with 200 instances per problem): N is the number of assets, T is the number of time steps, K is the number of units to be allocated at each time step and the maximum allowed holding (with $K' = K$), "encoding" refers to the method of encoding the integer problem into binary variables, "vars" is the number of binary variables required to encode the given problem, "density" is the density of the quadratic couplers, "qubits" is the number of physical qubits that were used, "chain" is the maximum number of physical qubits identified with a single binary variable, and $S(\alpha)$ refers to the success rate given a perturbation magnitude $\alpha\%$ (explained in the text).

N	T	K	encoding	vars	density	qubits	chain	$S(0)$	$S(1)$	$S(2)$
2	3	3	binary	12	0.52	31	3	100.00	100.00	100.00
2	2	3	unary	12	0.73	45	4	97.00	100.00	100.00
2	4	3	binary	16	0.40	52	4	96.00	100.00	100.00
2	3	3	unary	18	0.53	76	5	94.50	100.00	100.00
2	2	7	binary	12	0.73	38	4	90.50	100.00	100.00
2	5	3	binary	20	0.33	63	4	89.00	100.00	100.00
2	6	3	binary	24	0.28	74	4	50.00	100.00	100.00
3	2	3	unary	18	0.65	91	6	38.50	80.50	95.50
3	3	3	binary	18	0.45	84	5	35.50	80.50	96.50
3	4	3	binary	24	0.35	106	6	9.50	89.50	100.00

- The annealer solution is typically within a small and acceptable margin of error of the exact globally minimal solution.

Quantum-Ready Hierarchical Risk Parity

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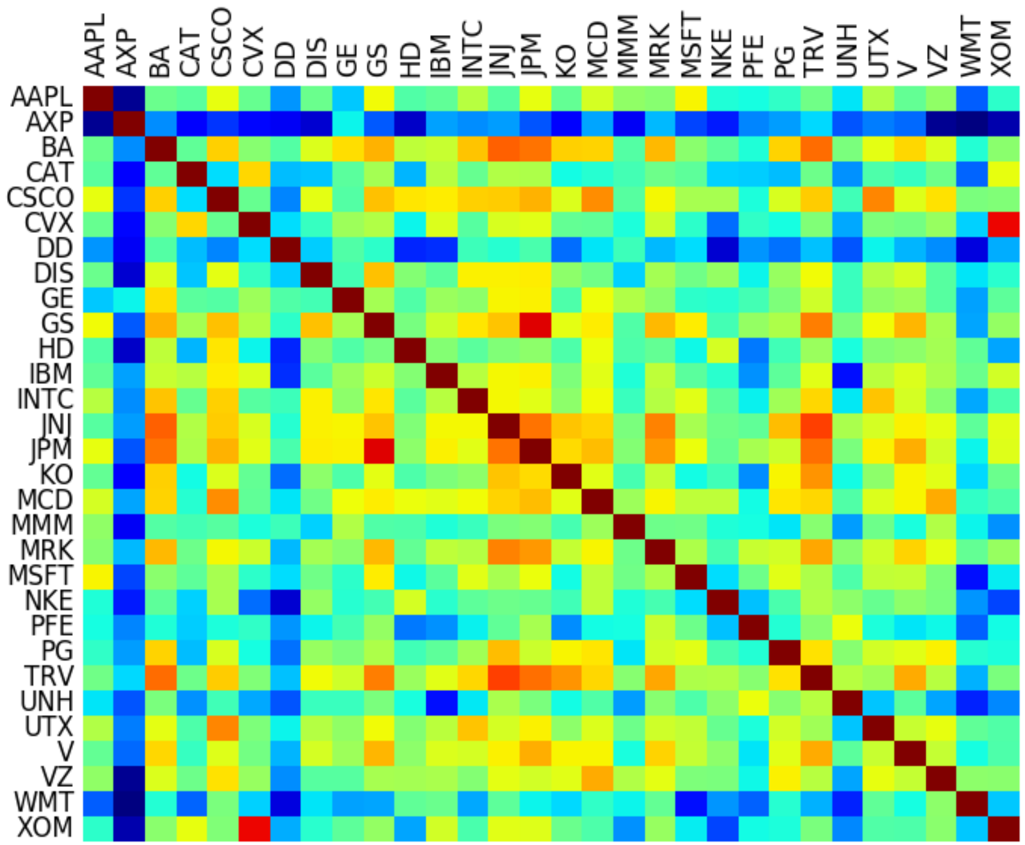
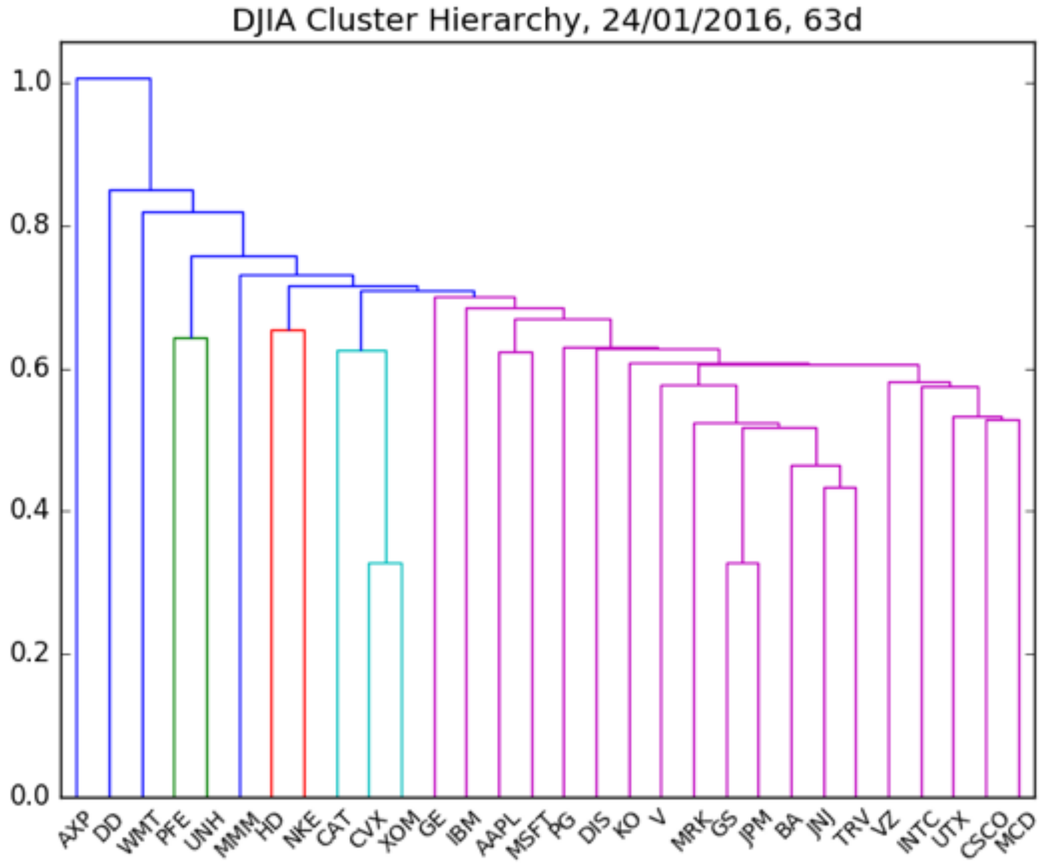
Quantum-Ready Hierarchical Risk Parity





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The Approach

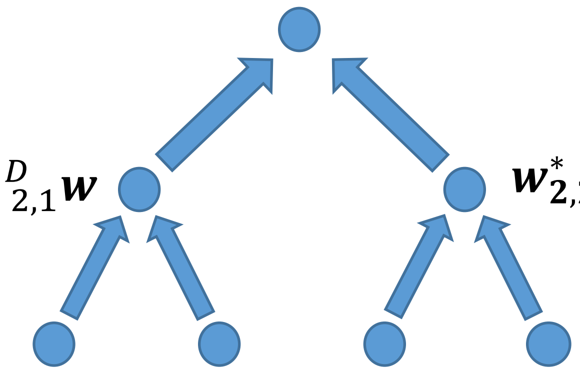
- Use tree structure to reduce connections between assets as well as estimation error
- Instead of minimizing variance via all correlations using all assets at once, solve the minimum variance problem at each node of the tree
- Build the final weight vector by recursively minimizing variance from the bottom of the tree to the top
- Ignore correlations in determining weights, use only in calculating variance of sub-trees
- Effect: Improve out-of-sample realized volatility
- Can also be applied to linear regression

Classical

$$\boldsymbol{w}^* = \arg\min_{\boldsymbol{w}} \boldsymbol{w}^T \widehat{\boldsymbol{\Sigma}} \boldsymbol{w} \;\; s.t. \sum_i w_i = 1$$

QHRP

$$\boldsymbol{w}_{\mathbf{1,1}}^* = \arg\min_{\boldsymbol{w}} \boldsymbol{w}^T \Sigma_{1,1}^D \boldsymbol{w}$$



$$\boldsymbol{w}_{\mathbf{2},\mathbf{1}}^* = \arg\min_{\boldsymbol{w}} \boldsymbol{w}^T \Sigma_{2,1}^D \boldsymbol{w}$$

$$\boldsymbol{w}_{\mathbf{2},\mathbf{2}}^* = \arg\min_{\boldsymbol{w}} \boldsymbol{w}^T \Sigma_{2,2}^D \boldsymbol{w}$$

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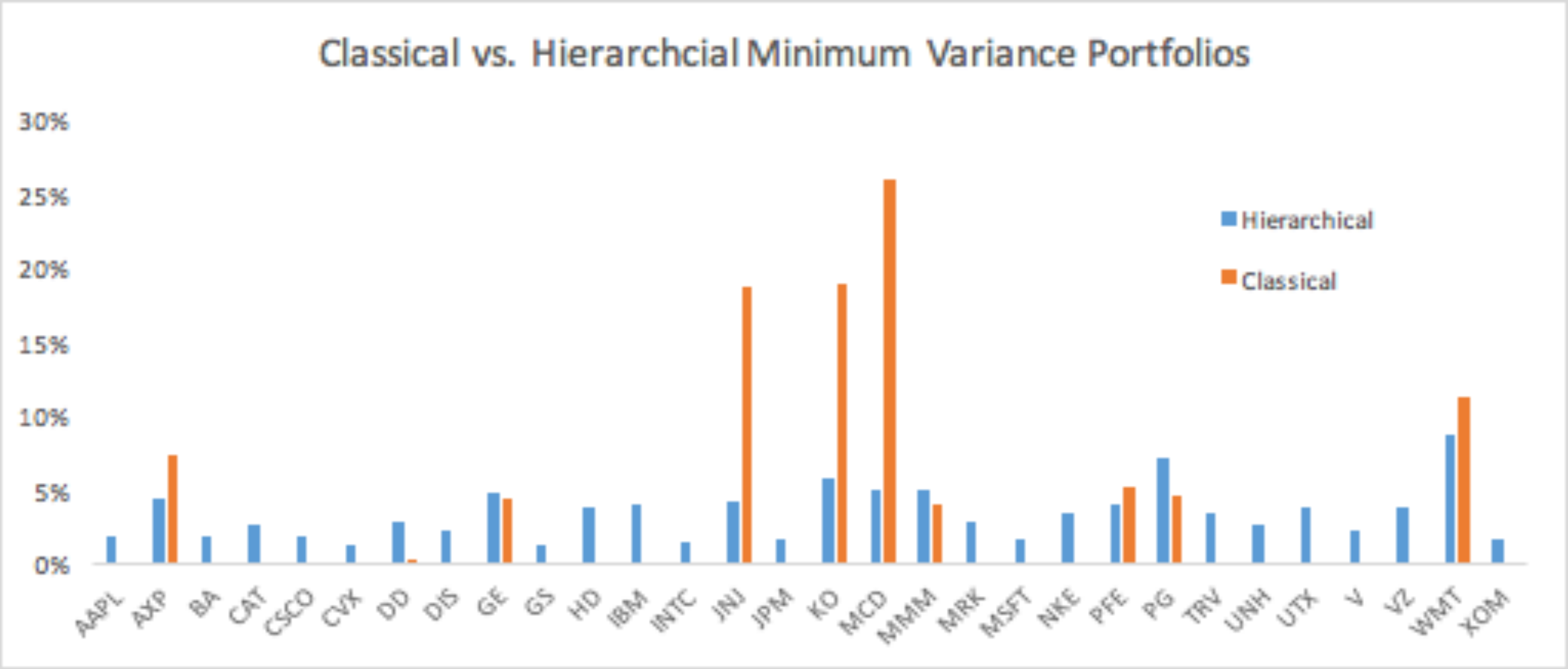
(Out-of-Sample) Minimum Risk Portfolios



- Classical algorithm minimizes **in-sample** risk, but sometimes missed the point
- In this example, it invests over 50% of the portfolio in just McDonalds, Coca-Cola and Johnson & Johnson
- QHRP provides more diversification

20%

Risk Improvement in Simulations

Out-of-sample volatility is reduced by 20% in simulated examples using 10 assets with 10% volatility each, random shocks, random correlations.

Real-Time Optimization Framework

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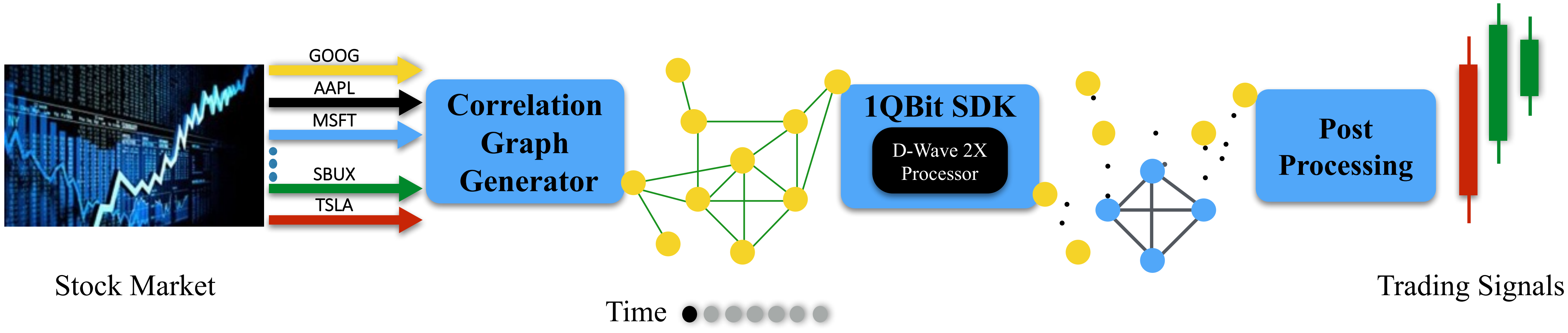
Real-Time Optimization Framework

- Stream Analytics
- Real-Time data signals
- Quantum powered analytics



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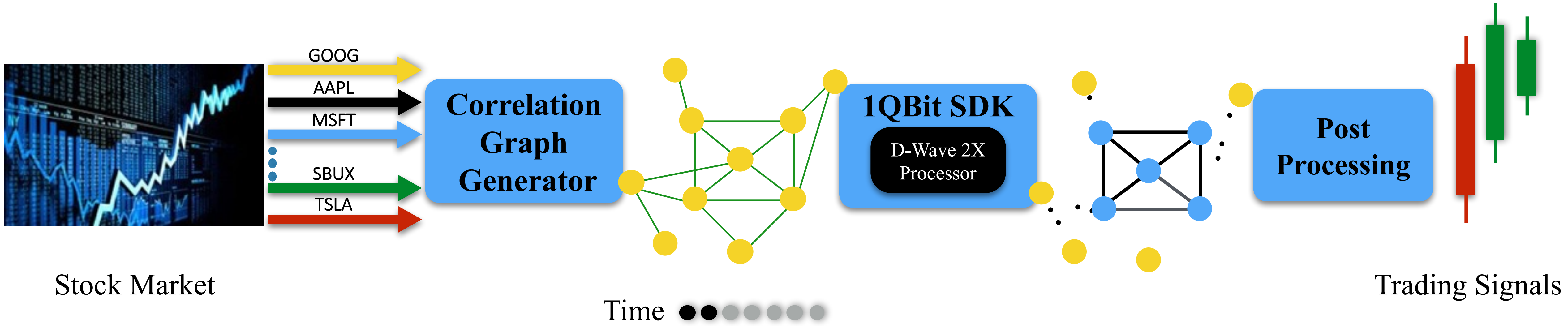

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Real-Time Optimization Framework

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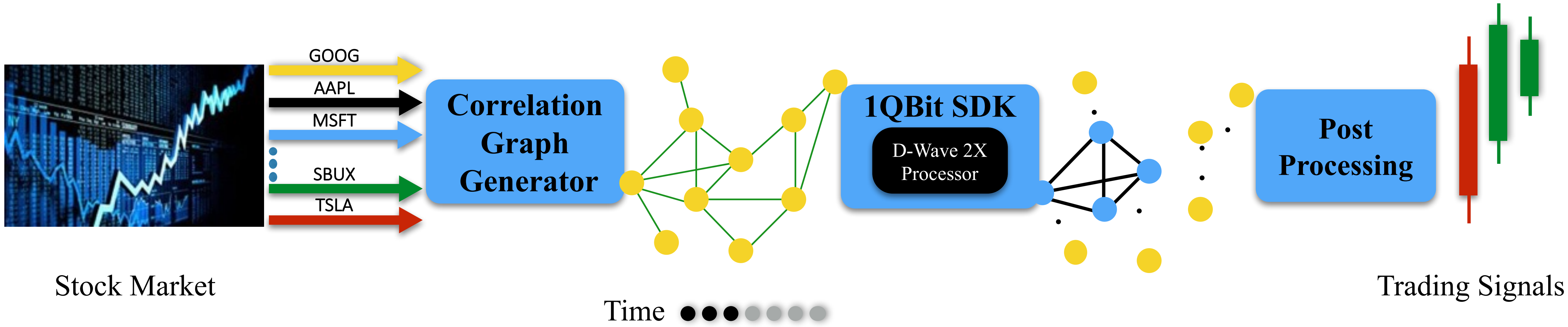
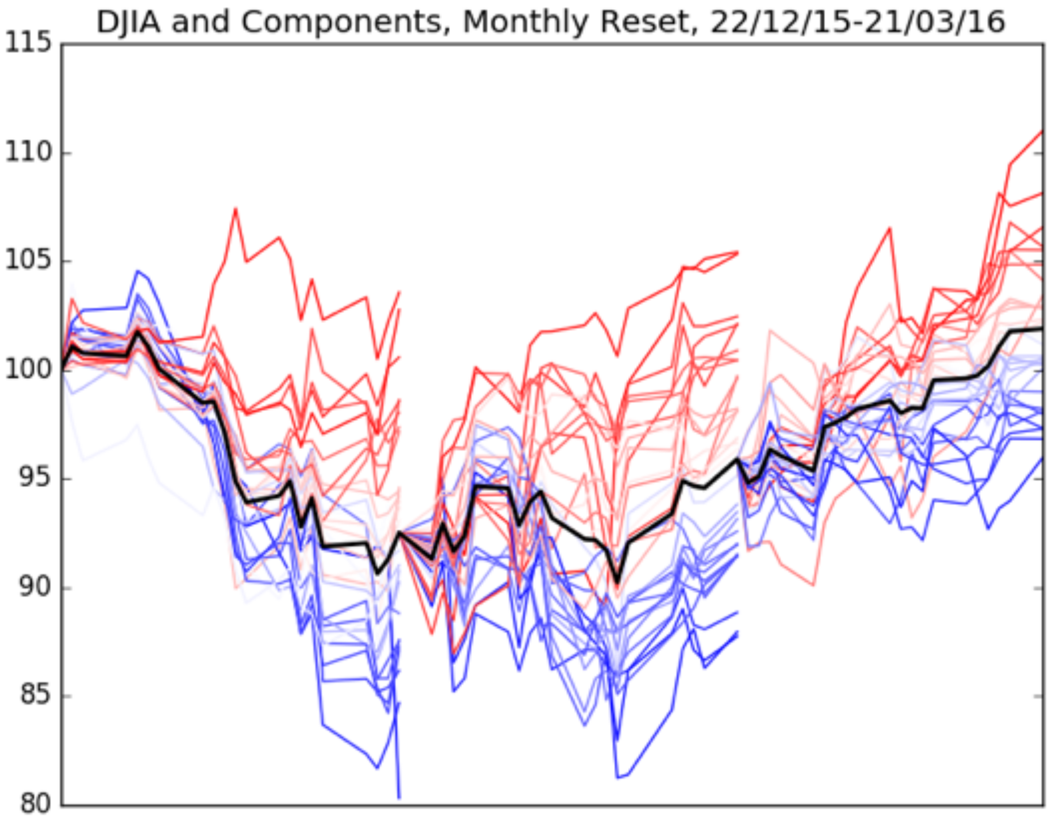


Tax Loss Harvesting

Tax Loss Harvesting

- Track an index using a subset of the components of the index while selling losses and moving into unowned components
- Integer Quadratic Programming: sell quantities from owned lots; buy new lots (avoiding wash sales)
- Offset gains and income; carry losses forward; create a tax buffer
- Quantum annealer used to find the optimal share quantities to buy and sell to generate the greatest tax benefit while staying within a specified tracking error.



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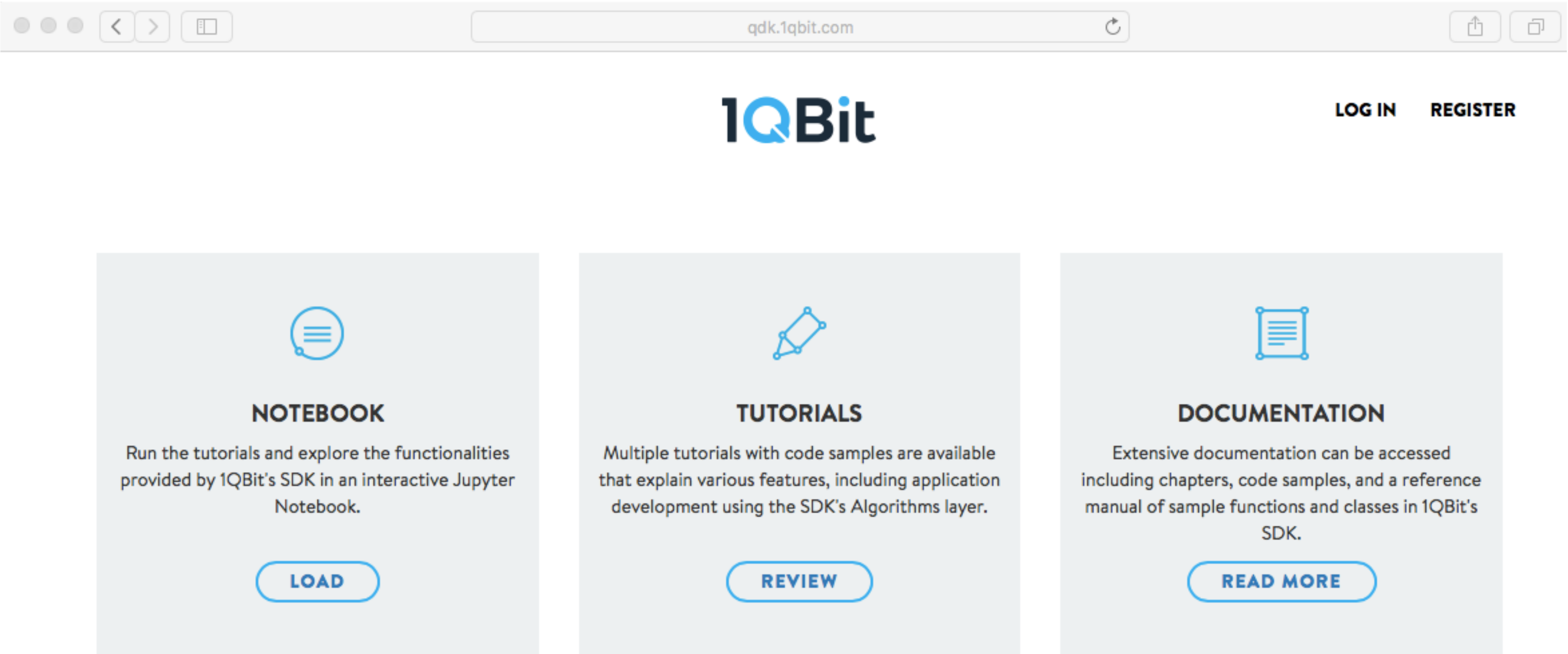
Tools and Resources

Tools

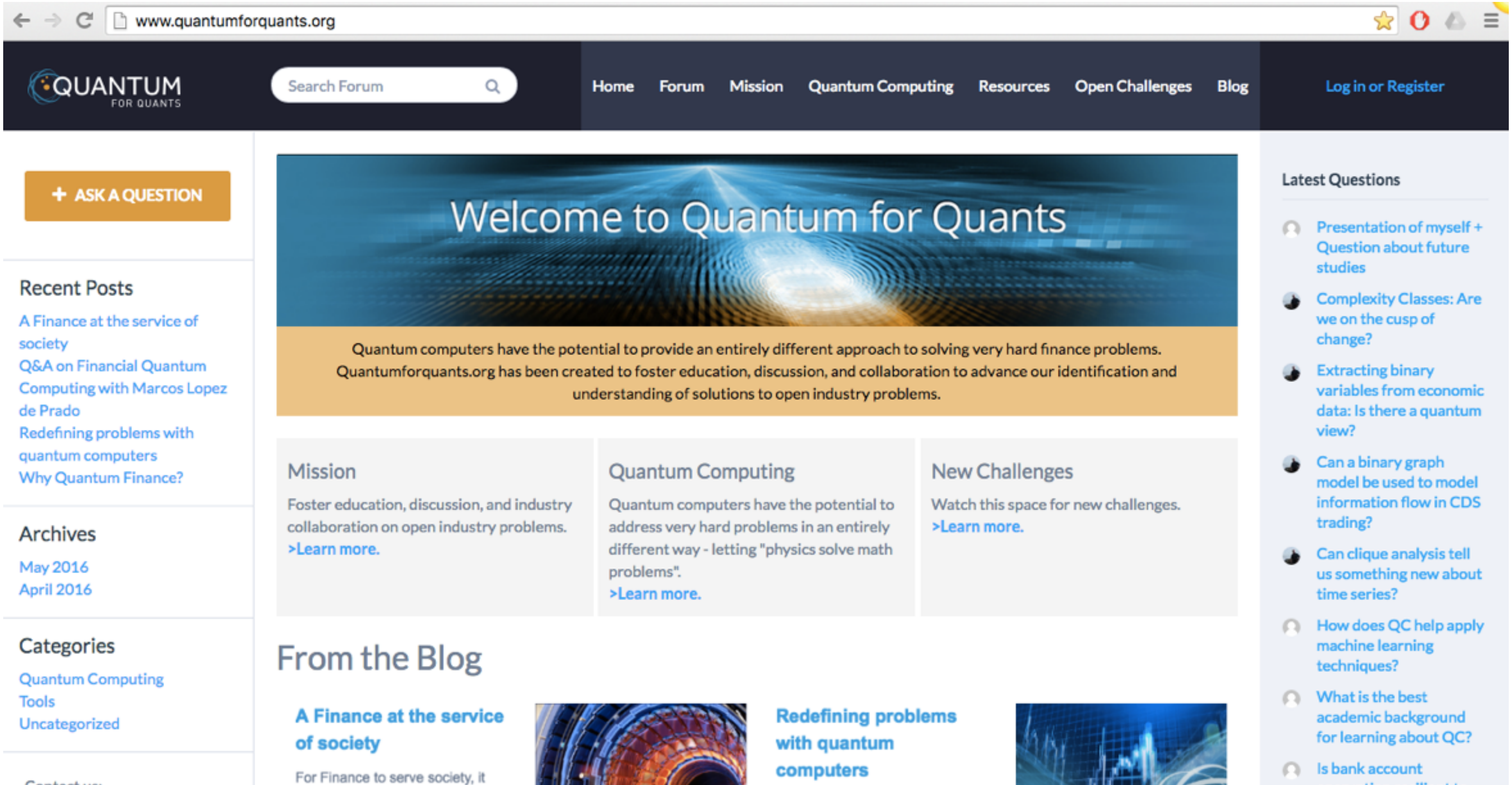
1QBit’s quantum-ready Software Development Kit:

Explore: 1qbit.com

Register for Beta Release: qdk.1qbit.com



Resources: www.quantumforquants.org



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